Class XII Session 2025-26
Subject - Mathematics
Sample Question Paper - 6

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If B $\begin{vmatrix} 1 & -2 \\ 1 & 4 \end{vmatrix} = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix}$ then matrix B is **[1]**

   a) I
   b) $\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix}$
   c) $\begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$
   d) $\begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix}$

2. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that A^{-1} = kA, then k equals **[1]**

   a) 19
   b) -1/19
   c) 1/19
   d) -19

3. A square matrix A is called singular if det. A is **[1]**

   a) 0
   b) Negative
   c) Positive
   d) Non–zero

4. If $y = \frac{\log x}{x}$, then $\frac{d^2y}{dx^2} =$ **[1]**

   a) $\frac{2\log x - 3}{x^4}$
   b) $\frac{2\log x + 3}{x^3}$
   c) $\frac{3 - 2\log x}{x^3}$
   d) $\frac{2\log x - 3}{x^3}$

5. If lines $\frac{2x-2}{2k} = \frac{4-y}{3} = \frac{z+2}{-1}$ and $\frac{x-5}{1} = \frac{y}{k} = \frac{z+6}{4}$ are at right angles, then the value of k is **[1]**

   a) -2
   b) 4
   c) 0
   d) 2

6. The general solution of the differential equation $\frac{dy}{dx}$ + y cot x = cosec x, is **[1]**

a) y sin x = x + C

b) y + x (sin x + cos x) = C

c) x + y sin x = C

d) x + y cos x = C

7. Minimize Z = 5x + 10 y subject to x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x, y ≥ 0 **[1]**

a) Minimum Z = 300 at (60, 0)

b) Minimum Z = 330 at (60, 0)

c) Minimum Z = 310 at (60, 0)

d) Minimum Z = 320 at (60, 0)

8. The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is equal to **[1]**

a) $\frac{\pi}{2}$

b) $\frac{7\pi}{2}$

c) $\frac{5\pi}{2}$

d) $\frac{3\pi}{2}$

9. $\int\frac{3x^2}{\sqrt{9-16x^6}}dx = ?$ **[1]**

a) $2\sin^{-1}\left(\frac{3x^3}{4}\right)+C$

b) $4\sin^{-1}\left(\frac{x^3}{4}\right)+C$

c) $\frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right)+C$

d) $\frac{1}{4}\sin^{-1}\left(\frac{x^3}{3}\right)+C$

10. If $A=\begin{bmatrix}2&3\\1&2\end{bmatrix}, B=\begin{bmatrix}1&3&2\\4&3&1\end{bmatrix}, C=\begin{bmatrix}1\\2\end{bmatrix}$ and $D=\begin{bmatrix}4&6&8\\5&7&9\end{bmatrix}$, then which of the following is defined? **[1]**

a) A + B

b) C + D

c) B + D

d) B + C

11. A feasible region of a system of linear inequalities is said to be ..., if it can be enclosed within a circle. **[1]**

a) In squared form

b) bounded

c) unbounded

d) in circled form

12. What is the angle which the vector $(\hat{i}+\hat{j}+\sqrt{2}\hat{k})$ makes with the z- axis? **[1]**

a) $\frac{\pi}{3}$

b) $\frac{\pi}{4}$

c) $\frac{2\pi}{3}$

d) $\frac{\pi}{6}$

13. Let A and B be two invertible matrices of order 3 × 3. If det (ABA') = 8 and det (AB^{-1}) = 8, then det ($BA^{-1}B'$) is equal to **[1]**

a) 1

b) $\frac{1}{16}$

c) 16

d) $\frac{1}{4}$

14. Three dice are thrown simultaneously. The probability of obtaining a total score of 5 is **[1]**

a) $\frac{1}{36}$

b) $\frac{5}{216}$

c) $\frac{1}{6}$

d) $\frac{1}{49}$

15. The solution of the differential equation $x\frac{dy}{dx}=y+x\tan\frac{y}{x}$, is **[1]**

a) $\sin\frac{y}{x}=cx$

b) $\sin\frac{x}{y}=x+C$

c) $\sin\frac{x}{y}=Cy$

d) $\sin\frac{y}{x}=Cy$

16. The value of $\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{i}\times\hat{k})+\hat{k}\cdot(\hat{i}\times\hat{j})$ is **[1]**

a) 1

b) 3

c) 0 | d) -1

17. Let f(x) = $\begin{cases} e^{1/x}, x<0 \\ x, x \geqslant 0 \end{cases}$, $then \; \underset{x \to 0}{Lt} \; f(x)$ **[1]**

a) does exist

b) is equal to non – zero real number

c) is equal to 0

d) does not exist

18. If (a_1, b_1, c_1) and (a_2, b_2, c_2) be the direction ratios of two parallel lines then **[1]**

a) $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$

b) $a_1^2 + b_1^2 + c_1^2 = a_2^2 + b_2^2 + c_2^2$

c) $a_1a_2 + b_1b_2 + c_1c_2 = 0$

d) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

19. **Assertion (A):** The function f(x) = x^2 - 4x + 6 is strictly increasing in the interval (2, ∞). **[1]**

Reason (R): The function f(x) = x^2 - 4x + 6 is strictly decreasing in the interval ($-\infty$, 2).

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The function f : R $\rightarrow$ R given by f(x) = x^3 is injective. **[1]**

Reason (R): The function f : X $\rightarrow$ Y is injective, if f(x) = f(y) $\Rightarrow$ x = y for all x, y $\in$ X.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the domain of f(x) = $\sin^{-1}(-x^2)$. **[2]**

OR

Which is greater, tan 1 or $\tan^{-1} 1$?

22. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing? **[2]**

23. Water is running into an inverted cone at the rate of π cubic metres per minute. The height of the cone is 10 metres, and the radius of its base is 5 m. How fast the water level is rising when the water stands 7.5 m below the base. **[2]**

OR

Prove that the function f given by f(x) = x^2 - x + 1 is neither strictly increasing nor strictly decreasing on (-1, 1).

24. Evaluate: $\int \tan^3 x \sec^2 x \, dx$ **[2]**

25. A matrix A of order 3 $\times$ 3 is such that |A| = 4. Find the value of |2A|. **[2]**

Section C

26. Find $\int \frac{x^2+1}{x^2-5x+6} dx$ **[3]**

27. An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two are drawn from first urn and put into the second urn and then a ball is drawn from the latter. Find the probability that it is a white ball. **[3]**

28. Prove that **[3]**

$\int\limits_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x+\cos x} dx = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1).$

OR

Evaluate: $\int \frac{(x-1)^2}{x^2+2x+2} dx$

29. Find a particular solution of the differential equation (x - y) (dx + dy) = dx - dy, given that y = -1, when x = 0. **[3]**

OR

Solve the differential equation $x\log|x|\frac{dy}{dx} + y = \frac{2}{x}\log|x|$.

30. If $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{b}$ in the form $\vec{b} = \vec{b}_1 + \vec{b}_2$, where $\vec{b}_1 \parallel \vec{a}$ and $\vec{b}_2 \perp \vec{a}$. **[3]**

OR

If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}, \vec{a}\times\vec{b} = \vec{a}\times\vec{c}, \vec{a} \neq \vec{0}$, then show that $\vec{b} = \vec{c}$.

31. If x = sin t, y = sin pt, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$. **[3]**

Section D

32. Find the area bounded by the curves y = x and y = x^3 **[5]**

33. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation. Write all the equivalence classes of R. **[5]**

OR

Let A = [-1, 1]. Then, discuss whether the following functions defined on A are one-one, onto or bijective:

i. $f(x) = \frac{x}{2}$
ii. g(x) = |x|
iii. h(x) = x|x|
iv. k(x) = x^2

34. Find X and Y, if $2x + 3y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3x + 2y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ **[5]**

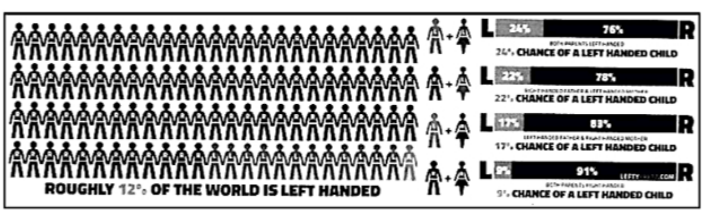
35. A wire of length 28m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined areas of the square and the circle is minimum? **[5]**

OR

If length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum.

Section E

36. Recent studies suggest that roughly 12% of the world population is left handed. **[4]**



Depending upon the parents, the chances of having a left handed child are as follows:

A. When both father and mother are left handed:
   Chances of left handed child is 24%.
B. When father is right handed and mother is left handed:
   Chances of left handed child is 22%.

C. When father is left handed and mother is right handed:

Chances of left handed child is 17%.

D. When both father and mother are right handed:

Chances of left handed child is 9%.

Assuming that P(A) = P(B) = P(C) = P(D) = $\frac{1}{4}$ and L denotes the event that child is left handed.

i. Find P $\left(\frac{L}{C}\right)$. (1)

ii. Find P $\left(\frac{L}{\bar{A}}\right)$. (1)

iii. Find P $\left(\frac{A}{L}\right)$. (2)

OR

Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Two motorcycles A and B are running at the speed more than allowed speed on the road along the lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} + \mu(2\hat{i} + \hat{j} + \hat{k})$, respectively.

i. Find the cartesian equation of the line along which motorcycle A is running. (1)

ii. Find the direction cosines of line along which motorcycle A is running. (1)

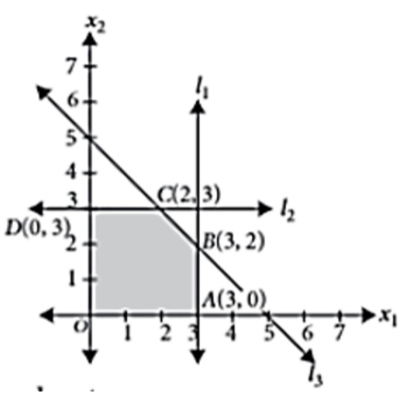
iii. Find the direction ratios of line along which motorcycle B is running. (2)

OR

Find the shortest distance between the given lines. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Corner points of the feasible region for an LPP are (0, 3), (5, 0), (6, 8), (0, 8). Let Z = 4x - 6y be the objective function.



i. At which corner point the minimum value of Z occurs? (1)

ii. At which corner point the maximum value of Z occurs? (1)

iii. What is the value of (maximum of Z - minimum of Z)? (2)

OR

The corner points of the feasible region determined by the system of linear inequalities are (2)

Solution

Section A

1.

(b) $\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix}$

Explanation:

Let B = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$\begin{vmatrix} a & b \\ c & d \end{vmatrix}\begin{vmatrix} 1 & -2 \\ 1 & -4 \end{vmatrix} = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix}$

$\begin{vmatrix} a+b & -2a+4b \\ c+d & -2c+4d \end{vmatrix} = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix}$

Now, comparing the corresponding dement

a + b = 6 ...(i)

-2a + 4b = 0

-2a = -4b

a = 2b ...(ii)

c + d = 0 ...(iii)

-2c + 4d = 6

putting (ii) in (i)

3b = 6

b = 2

∴ a = 4

Now, from equation (iii) and (iv)

-2c + 4(-c) = 6

-6c = 6

c = -1

∴ d = 1

matrix B = $\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix}$

2.

(c) 1/19

Explanation:

$A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

Using adjoint matrix

$A^{-1} = \frac{-1}{19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \frac{1}{19}A$

$k = \frac{1}{19}$

3. **(a)** 0

Explanation:

For a singular matrix, |A| = 0.

4.

(d) $\frac{2\log x - 3}{x^3}$

Explanation:

$\frac{d}{dx}\left(\frac{\log x}{x}\right)=\frac{x.\frac{1}{x}-\log x.1}{x^2}=\frac{1-\log x}{x^2}$

$\Rightarrow \frac{d}{dx}\left(\frac{1-\log x}{x^2}\right)$

$=\frac{x^2.(-\frac{1}{x})-(1-\log x).2x}{x^4}=\frac{-x-2x+2x\log x}{x^4}=\frac{x(2\log x-3)}{x^4}=\frac{2\log x-3}{x^3}$

5. **(a)** -2

Explanation:

Given lines are $\frac{2x-2}{2k}=\frac{4-y}{3}=\frac{z+2}{-1}$ and $\frac{x-5}{1}=\frac{y}{k}=\frac{z+6}{4}$

Writing the above equation in standard form, we get

$\Rightarrow \frac{2(x-1)}{2k}=\frac{-(y-4)}{3}=\frac{z+2}{-1}$

$\Leftrightarrow \frac{(x-1)}{k}=\frac{y-4}{-3}=\frac{z+2}{-1}$

Now, the direction ratio of the first line is (k, -3, -1) and the direction ratio of second line is (1, k, 4)

Since, lines are perpendicular,

$\therefore$ (k $\times$ 1) + (-3 $\times$ k) + (-1 $\times$ 4) = 0

$\Rightarrow$ k - 3k - 4 = 0

$\Rightarrow$ -2k - 4 = 0

$\therefore$ k = -2

6. **(a)** y sin x = x + C

Explanation:

We have,

$\frac{dy}{dx}$ + y cot x = cosec x

Comparing with $\frac{dy}{dx}$ + Py = Q of the above equation then, we get

$\Rightarrow$ P = cot x, Q = cosec x

I.F. = $e^{\int Pdx}=e^{\int \cot x dx}=e^{\log \sin x}$ = sin x

Multiplying on both sides by sin x

sin x $\frac{dy}{dx}$ + y cos x = 1

$\Rightarrow \frac{d}{dx}(y\sin x)$ = 1

$\Rightarrow$ y sin x = $\int 1dx$

$\Rightarrow$ y sin x = x + C

7. **(a)** Minimum Z = 300 at (60, 0)

Explanation:

Objective function is Z = 5x + 10 y ……………………(1).

The given constraints are : x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x, y ≥ 0 .

The corner points are obtained by drawing the lines x+2y =120, x+y = 60 and x-2y = 0. The points so obtained are (60,30), (120,0), (60,0) and (40,20)

Corner points	Z = 5x + 10y
D(60 ,30)	600
A(120,0)	600
B(60,0)	300……………………..(Min.)
C(40,20)	400

Here , Z = 300 is minimum at (60, 0).

8. **(a)** $\frac{\pi}{2}$

Explanation:

We have, $\cos^{-1}$ cos $\frac{3\pi}{2}$

We know that,

cos $\frac{3\pi}{2}$ = 0

So, $\cos^{-1}$ cos $\frac{3\pi}{2}$ = $\cos^{-1}$ 0

Let, $\cos^{-1} 0 = \theta$

$\Rightarrow \cos\theta = 0$

Principal value of $\cos^{-1} x$ is $[0, \pi]$

For, $\cos\theta = 0$

So, $\theta = \frac{\pi}{2}$

9.

(c) $\frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right) + C$

Explanation:

Put $x^3 = t$ and $3x^2\,dx = dt$

$\therefore I = \int \frac{dt}{\sqrt{9-16t^2}} = \frac{1}{4}\int \frac{dt}{\sqrt{\frac{9}{16}-t^2}} = \frac{1}{4}\cdot\int \frac{dt}{\sqrt{\left(\frac{3}{4}\right)^2-t^2}}$

$= \frac{1}{4}\sin^{-1}\frac{t}{(3/4)} + C = \frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right) + C$

10.

(c) B + D

Explanation:

Only B + D is defined because matrices of the same order can only be added.

11.

(b) bounded

Explanation:

A feasible region of a system of linear inequalities is said to be bounded, if it can be enclosed within a circle.

12.

(b) $\frac{\pi}{4}$

Explanation:

$\frac{\pi}{4}$

Hint

$\vec{a} = (\hat{i} + \hat{j} + \sqrt{2}\hat{k}) \Rightarrow |\vec{a}| = \ |\vec{a}| = \sqrt{1^2 + 1^2 + (\sqrt{2})^2} = \sqrt{4} = 2$

Direction ratios of $\vec{a}\ are\ (1, 1, \sqrt{2})$

Direction cosines of $\vec{a}$ are $\left(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}\right)$, i.e., $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

Direction coines along to z-axis(0,0,1)

$\therefore \cos\gamma = \frac{1}{\sqrt{2}} \Rightarrow \gamma = \frac{\pi}{4}$

13.

(b) $\frac{1}{16}$

Explanation:

$\frac{1}{16}$

14. **(a)** $\frac{1}{36}$

Explanation:

$\frac{1}{36}$

15. **(a)** $\sin\frac{y}{x} = cx$

Explanation:

We have ,

$x\frac{dy}{dx} = y + x\tan\left(\frac{y}{x}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$ .....(I)

Put $\frac{y}{x}$ = v

y = vx

Differentiating on both sides,

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + \tan v = v + x\frac{dv}{dx}$ ...from (i)

$\tan v = x\frac{dv}{dx}$

$\frac{dx}{x} = \cot v dv$

$\int \frac{dx}{x} = \int \cot v dv$

log |x| + log C = log |sin v|

Cx = sin v

$\sin\left(\frac{y}{x}\right)$ = c x

16. **(a)** 1

Explanation:

Given: $(\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k}(\hat{i} \times \hat{j})$

i. $(\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ = $\hat{\mathrm{i}} . \hat{\mathrm{I}} + \hat{\mathrm{j}} \cdot (-\hat{\mathrm{j}}) + \hat{\mathrm{k}}\hat{\mathrm{k}}$

$= 1 - \hat{j} . \hat{j} + 1$

=1 - 1 + 1

=1

17.

(c) is equal to 0

Explanation:

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} e^{\frac{1}{x}} = 0, \quad \lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} x = 0 \therefore \lim\limits_{x \to 0} f(x) = 0$

18.

(d) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Explanation:

We know that if there are two parallel lines then their direction ratios must have a relation

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

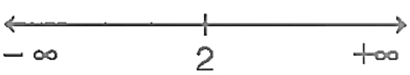
19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

We have, f(x) = x^2 - 4x + 6

or f'(x) = 2x - 4 = 2 (x - 2)



Therefore, f'(x) = 0 gives x = 2.

Now, the point x = 2 divides the real line into two disjoint intervals namely, $(-\infty, 2)$ and $(2, \infty)$.

In the interval $(-\infty, 2)$, f'(x) = 2x - 4 < 0.

Therefore, f is strictly decreasing in this interval.

Also, in the interval $(2, \infty)$, f'(x) > 0 and so the function f is strictly increasing in this interval.

Hence, both the statements are true but Reason is not the correct explanation of Assertion.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Assertion: Here, f : R $\to$ R is given as

f(x) = x^3.

Suppose f(x) = f(y)

where x, y $\in$ R

$\Rightarrow$ $x^3 = y^3$ ...(i)

Now, we try to show that x = y

Suppose x $\neq$ y, their cubes will also not be equal.

$x^3 \neq y^3$

However, this will be a contradiction to Eq. (i).

Therefore, x = y. Hence, f is injective. Hence, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

Section B

21. The domain of $\sin^{-1} x$ is [-1,1]. Therefore, $f(x) = \sin^{-1}(-x^2)$ is defined for all x satisfying $-1 \leq -x^2 \leq 1$

$\Rightarrow \quad 1 \geq x^2 \geq -1$

$\Rightarrow 0 \leq x^2 \leq 1$

$\Rightarrow x^2 \leq 1$

$\Rightarrow x^2 - 1 \leq 0$

$\Rightarrow (x - 1)(x + 1) \leq 0$

$\Rightarrow -1 \leq x \leq 1$

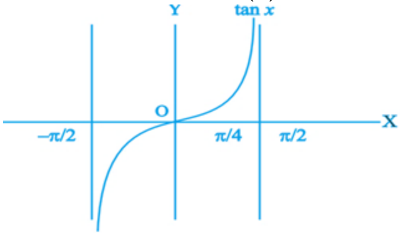
Hence, the domain of $f(x) = \sin^{-1}(-x^2)$ is [-1, 1].

OR

From Fig. we note that tan x is an increasing function in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, since $1 > \frac{\pi}{4} \Rightarrow \tan 1 > \tan \frac{\pi}{4}$. This gives tan 1 > 1

$\Rightarrow \tan 1 > 1 > \frac{\pi}{4}$

$\Rightarrow \tan 1 > 1 > \tan^{-1}(1)$



22. Let x cm be the radius and y be the enclosed area of the circular wave at any time t.

Rate of increase of radius of circular wave = 5 cm/sec

$\Rightarrow \frac{dx}{dt}$ is positive and = 5 cm/sec

$\Rightarrow \frac{dx}{dt} = 5cm/\sec$ ...(i)

$y = \pi x^2$

$\therefore$ Rate of change of area $= \frac{dy}{dt} = \pi \frac{d}{dt} x^2$

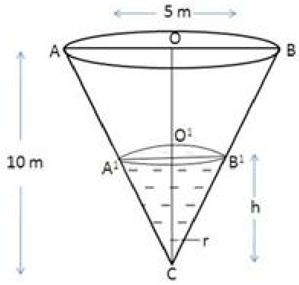
$= \pi.2x \frac{dx}{dt} = 2\pi x\,(5)$ (from (i)

$= 10\pi x cm^2/\sec$

Putting $x = 8cm$ (given),

$\frac{dy}{dt} = 10\pi\,(8) = 80\pi cm^2/\sec$

Since $\frac{dy}{dt}$ is positive, therefore area of circular wave is increasing at the rate of $80\pi cm^2/\sec$.

23. 

Let α be the semi vertical angle of the cone whose height CO = 10 m and radius OB = 5 m.

Now, $\tan \alpha = \frac{OB}{CO} = \frac{5}{10}$

$\tan \alpha = \frac{1}{2}$

Let V be the volume of water in the cone, then

$v = \frac{1}{3}\pi(0'B')^2\,(CO')$

$v = \frac{1}{3}\pi h^3 \tan^2 \alpha$

$v=\frac{\pi}{12}h^3$
$\frac{dv}{dt}=\frac{3\pi}{12}h^2\frac{dh}{dt}$
$\pi=\frac{\pi}{4}h^2\frac{dh}{dt}$
$\frac{dh}{dt}=\frac{4}{h^2}$
$\left(\frac{dh}{dt}\right)_{h\ =\ 2.5}=\frac{4}{(2.5)^2}$
$=\frac{4}{6.25}$
= 0.64m/min.

So, the water level is rising at the rate of 0.64 m/min.

OR

Given: $f(x)=x^2-x+1$ f(x) = x^2 - x + 1

⇒ f'(x) = 2x - 1

f(x) is strictly increasing if f'(x) < 0

⇒2x - 1 > 0

$\Rightarrow x>\frac{1}{2}$

i.e., increasing on the interval $\left(\frac{1}{2},1\right)$

f(x) is strictly decreasing if f'(x) < 0

⇒ 2x - 1 < 0

$\Rightarrow x<\frac{1}{2}$

i.e., decreasing on the interval $\left(-1,\frac{1}{2}\right)$

hence, f(x) is neither strictly increasing nor decreasing on the interval (-1, 1).

24. Let I = $\int \tan^3 x \sec^2 x\, dx$

Now let tan x = t. Then, d(tan x) = dt ⇒ $\sec^2 x\, dx$ = dt ⇒ $dx=\frac{dt}{\sec^2 x}$

Put tan x = f and dx = dt $\sec^2$ x, we get

I = $\int \tan^3 x \sec^2 x\, dx$

$=\int t^3\sec^2 x\times\frac{dt}{\sec^2 x}$

$=\int t^3 dt$

$=\frac{t^4}{4}+C$

$=\frac{\tan^4 x}{4}+C$

25. We are given that,

Order of matrix A = 3

|A| = 4

We need to find the value of |2A|.

By the property of determinant of a matrix,

$|KA| = K^n|A|$

Where the order of the matrix A is n.

Similarly,

$|2A| = 2^3|A|$ ...[∵ Order of matrix A = 3]

⇒ |2A| = 8|A|

Substituting the value of |A| in the above equation,

⇒ |2A| = 8 × 4

⇒ |2A| = 32

Thus, the value of |2A| is 32.

Section C

26. Here the integrand $\frac{x^2+1}{x^2-5x+6}$ is not proper rational function, so we divide x^2 + 1 by x^2 - 5x + 6 and find that

$\frac{x^2+1}{x^2-5x+6}=1+\frac{5x-5}{x^2-5x+6}=1+\frac{5x-5}{(x-2)(x-3)}$

Let $\frac{5x-5}{(x-2)(x-3)}=\frac{A}{x-2}+\frac{B}{x-3}$

So that 5x - 5 = A (x - 3) + B (x - 2)

Equating the coefficients of x and constant terms on both sides, we get A + B = 5 and 3A + 2B = 5.

Solving these equations, we get A = -5 and B = 10

Thus, $\frac{x^2+1}{x^2-5x+6} = 1 - \frac{5}{x-2} + \frac{10}{x-3}$

Therefore, $\int \frac{x^2+1}{x^2-5x+6} dx = \int dx - 5\int \frac{1}{x-2} dx + 10\int \frac{dx}{x-3}$

= x - 5 log | x - 2| + 10 log | x - 3| + C.

27. A white ball can be drawn in three mutually exclusive ways:

i. By transferring two black balls from first to second urn, then drawing a white ball
ii. By transferring two white balls from first to second urn, then drawing a white ball
iii. By transferring a white and a black ball from first to second urn, then drawing a white ball

Consider the following events:

E_1 = Two black balls are transferred from first to second bag

E_2 = Two white balls are transferred from first to second bag

E_2 = A white and a black ball is transferred from first to second bag

A = A white ball is drawn

Therefore, we have,

$P(E_1) = \frac{^3C_2}{^{13}C_2} = \frac{3}{78}$

$P(E_2) = \frac{^{10}C_2}{^{13}C_2} = \frac{45}{78}$

$P(E_3) = \frac{^{10}C_1 \times {^3C_1}}{^{13}C_2} = \frac{30}{78}$

Now,

$P\left(\frac{A}{E_1}\right) = \frac{3}{10}$

$P\left(\frac{A}{E_2}\right) = \frac{5}{10}$

$P\left(\frac{A}{E_3}\right) = \frac{4}{10}$

Using the law of total probability, we get

Required probability = P(A) = $P(E_1)P(\frac{A}{E_1}) + P(E_2)P(\frac{A}{E_2}) + P(E_3)\,P(\frac{A}{E_3})$

$= \frac{3}{78} \times \frac{3}{10} + \frac{45}{78} \times \frac{5}{10} + \frac{30}{78} \times \frac{4}{10}$

$= \frac{9}{780} + \frac{225}{780} + \frac{120}{780}$

$= \frac{354}{780} = \frac{59}{130}$

28. According to the question, $I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$ ...(i)

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$

$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos x + \sin x} dx$ ...(ii)

Adding Equations (i) and (ii),

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x} dx$

$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} dx$

$[\because sin^2 x + cos^2 x = 1\,]$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{1}{\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}} dx$

$\left[\because \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} \text{ and } \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right]$

$= \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}dx$

Put $\tan\frac{x}{2} = t \Rightarrow \sec^2\frac{x}{2}\cdot\frac{1}{2}dx = dt \Rightarrow \sec^2\frac{x}{2}dx = 2dt$

Lower limit when x = 0, then t = tan 0 = 0

Upper limit when x = $\frac{\pi}{2}$, then t = tan $\frac{\pi}{4} = 1$.

$\therefore \quad 2I = \int_0^1 \frac{2dt}{2t+1-t^2}dt = 2\int_0^1 \frac{dt}{-[t^2-2t-1]}dt$

$= 2\int_0^1 \frac{dt}{-\left[(t-1)^2-1-1\right]}dt = 2\int_0^1 \frac{dt}{(\sqrt{2})^2-(t-1)^2}$

$= \left[\frac{2}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+t-1}{\sqrt{2}-t+1}\right|\right]_0^1$

$\left[\because \int\frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C\right]$

$= \frac{1}{\sqrt{2}}\left[\log\frac{\sqrt{2}+1-1}{\sqrt{2}-1+1} - \log\frac{\sqrt{2}+0-1}{\sqrt{2}-0+1}\right]$

$= \frac{1}{\sqrt{2}}\left[\log 1 - \log\frac{\sqrt{2}-1}{\sqrt{2}+1}\right]$

$= -\frac{1}{\sqrt{2}}\log\left[\frac{\sqrt{2}-1}{\sqrt{2}+1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}\right]$ [$\because$log 1 = 0]

$= \frac{-1}{\sqrt{2}}\log\frac{2-1}{(\sqrt{2}+1)^2}$ [$\because$(a - b) (a + b) = $a^2 - b^2$]

$= \frac{-1}{\sqrt{2}}\log\frac{1}{(\sqrt{2}+1)^2}$

$\Rightarrow \quad 2I = \frac{2}{\sqrt{2}}\log(\sqrt{2}+1) \Rightarrow I = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$

OR

Let I = $\int\frac{(x-1)^2}{x^2+2x+2}dx$

$= \int\left(\frac{x^2-2x+1}{x^2+2x+2}\right)dx$

Therefore by long division we have,

$$\begin{array}{r|l} & \quad 1 \\ x^2+2x+2 & x^2-2x+1 \\ & x^2+2x+2 \\ & - \quad - \quad\quad - \\ \hline & -4x-1 \\ \hline \end{array}$$

Therefore,

$\frac{x^2-2x+1}{x^2+2x+2} = 1 - \frac{(4x+1)}{x^2+2x+2}$ ...(i)

Let 4x + 1 = $A\frac{d}{dx}$(x^2 + 2x + 2) + B

4x + 1 A (2x + 2) + B

4x + 1 (2A)x + 2A + B

Equating Coefficients of like terms

2A = 4

A = 2

2A + B = 1

2 × 2 + B = 1

B = -3

$\int\left(\frac{x^2-2x+1}{x^2+2x+2}\right)dx$

= $\int$ dx - 2$\int\frac{(2x+2)}{x^2+2x+2}dx + 3\int\frac{dx}{x^2+2x+2}$

= $\int$ dx - 2 $\int\frac{(2x+2)}{x^2+2x+2}dx + 3\int\frac{dx}{(x+1)^2+1^2}$

= x - 2 log |x + 2x + 2| + $\frac{3}{1}\tan^{-1}\left(\frac{x+1}{1}\right) + C$

= x - 2 log |x + 2x + 2| + 3 $\tan^{-1}$ (x + 1) + C

29. It is given that (x - y)(dx + dy) = dx - dy

⇒ (x - y + 1)dy = (1 - x + y)dx

$\Rightarrow \frac{dy}{dx} = \frac{1-x+y}{x-y+1}$ ......(i)

Let x – y = t

$\Rightarrow \frac{d}{dx}(x-y) = \frac{dt}{dx}$

$\Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$

Now, let us substitute the value of x-y and $\frac{dy}{dx}$ in equation (i), we get,

$1 - \frac{dt}{dx} = \frac{1-t}{1+t}$

$\Rightarrow \frac{dt}{dx} = 1 - \left(\frac{1-t}{1+t}\right)$

$\Rightarrow \frac{dt}{dx} = \frac{(1+t)-(1-t)}{1+t}$

$\Rightarrow \frac{dt}{dx} = \frac{2t}{1+t}$

$\Rightarrow \left(\frac{1+t}{t}\right) dt = 2dx$

$\Rightarrow \left(1 + \frac{1}{t}\right) dt = 2dx$ .....(ii)

On integrating both side, we get,

t + log|t| = 2x + C

⇒ (x - y) + log |x - y| = 2x + C

⇒ log|x – y| = x + y + C ....(iii)

Now, y = -1 at x = 0

Then, equation (iii), we get,

log 1 = 0 - 1 + C

⇒ C = 1

Substituting C = 1 in equation (iii), we get,

log|x - y| = x + y + 1

Therefore, a particular solution of the given differential equation is log|x - y| = x + y + 1.

OR

We have to solve,

$x \log|x| \frac{dy}{dx} + y = \frac{2}{x}\log|x|$

On dividing both sides by x log x, we get

$\frac{dy}{dx} + \frac{y}{x\log|x|} = \frac{2}{x^2}\frac{\log|x|}{\log|x|} = \frac{2}{x^2}$

which is a linear differential equation of first order, which is of the form of $\frac{dy}{dx} + Py = Q$,

Here, $P = \frac{1}{x\log|x|}$ and $Q = \frac{2}{x^2}$

We know that ,

$\text{IF} = e^{\int Pdx} = e^{\int \frac{1}{x\log|x|}dx}$

put $\log|x| = t \Rightarrow \frac{1}{x}dx = dt$

$\therefore IF = \int \frac{1}{t}dt = \log|t| = \log|\log x|$

$\Rightarrow \quad \text{IF} = \log|x| \quad [\because e^{\log x} = x]$

Now, solution of above equation is given by

$y \times \text{IF} = \int (Q \times \text{IF})dx + C$

$y\log|x| = \int \underset{II}{\frac{2}{x^2}} \underset{I}{\log|x|}dx + C$

$\Rightarrow \quad y\log|x| = 2\left[\log|x|\int \frac{1}{x^2}dx - \int\left(\frac{d}{dx}(\log|x|)\cdot\int\frac{1}{x^2}dx\right)dx\right] + C$ [using integration by parts]

$\Rightarrow \quad y\log|x| = 2\left[\log|x|\cdot\left(-\frac{1}{x}\right) - \int \frac{1}{x}\cdot\left(-\frac{1}{x}\right)dx + C\right.$

$\Rightarrow \quad y\log|x| = 2\left[-\frac{1}{x}\log|x| + \int \frac{1}{x^2}dx\right] + C$

$\therefore \quad y\log|x| = -\frac{2}{x}\log|x| - \frac{2}{x} + C$

30. According to the question,

$\vec{a} = 3\hat{i} - \hat{j}$ and

$\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$

Let $\overrightarrow{b_1} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and

$\overrightarrow{b_2} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$\overrightarrow{b_1} + \overrightarrow{b_2} = \vec{b}, \overrightarrow{b_1} \| \vec{a}$ and
$\overrightarrow{b_2} \perp \vec{a}$.

Consider, $\overrightarrow{b_1} + \overrightarrow{b_2} = \vec{b}$

$\Rightarrow \quad (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j} + (z_1 + z_2)\hat{k} = 2\hat{i} + \hat{j} - 3\hat{k}$

On comparing the coefficient of $\hat{i}, \hat{j}$ and $\hat{k}$ both sides; we get

$\Rightarrow x_1 + x_2 = 2$ ...(i)

$y_1 + y_2 = 1$ ...(ii)

and $z_1 + z_2 = -3$ ...(iii)

Now, consider $\overrightarrow{b_1} \| \vec{a}$

$\Rightarrow \quad \frac{x_1}{3} = \frac{y_1}{-1} = \frac{z_1}{0} = \lambda(\text{say})$

$\Rightarrow \quad x_1 = 3\lambda, y_1 = -\lambda$ and $z_1 = 0$ ...(iv)

On substituting the values of x,y and z, form Eq. (iv) to Eq. (i), (ii) and (iii), respectively, we get

$x_2 = 2 - 3\lambda, y_2 = 1 + \lambda$ and $z_2 = -3$ ...(v)

Since, $\overrightarrow{b_2} \perp \vec{a}$, therefore $\vec{b_2} \cdot \vec{a} = 0$

$\Rightarrow \quad 3x_2 - y_2 = 0$

$\Rightarrow \quad 3(2 - 3\lambda) - (1 + \lambda) = 0$

$\Rightarrow \quad 6 - 9\lambda - 1 - \lambda = 0$

$\Rightarrow \quad 5 - 10\lambda = 0 \Rightarrow \lambda = \frac{1}{2}$

On substituting $\lambda = \frac{1}{2}$ in Eqs. (iv) and Eqs. (iv) and (v), we get

$x_1 = \frac{3}{2}, y_1 = \frac{-1}{2}, z_1 = 0$

and $x_2 = \frac{1}{2}, y_2 = \frac{3}{2}$ and $z_2 = -3$

Hence, $\vec{b} = \vec{b_1} + \vec{b_2} = \left(\frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}\right) + \left(\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}\right)$

$= 2\hat{i} + \hat{j} - 3\hat{k}$

OR

Given ,

$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \neq \overrightarrow{0}$

$\Rightarrow \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} = 0$ and $\vec{a} \neq \overrightarrow{0}$

$\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$ and $\vec{a} \neq \overrightarrow{0}$

$\Rightarrow \vec{b} - \vec{c} = \overrightarrow{0}$ or $\vec{a} \perp (\vec{b} - \vec{c})$ $[\because \vec{a} \neq \overrightarrow{0}]$

$\Rightarrow \vec{b} = \vec{c}$ or, $\vec{a} \perp (\vec{b} - \vec{c})$ ...(i)

Again given,

$\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ and $\vec{a} \neq \overrightarrow{0}$

$\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \overrightarrow{0}$ and $\vec{a} \neq \overrightarrow{0}$

$\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \overrightarrow{0}$ and $\vec{a} \neq \overrightarrow{0}$

$\Rightarrow \vec{b} - \vec{c} = \overrightarrow{0}$ or $\vec{a} \| (\vec{b} - \vec{c})$ $[\because \vec{a} \neq \overrightarrow{0}]$

$\Rightarrow \vec{b} = \vec{c}$ or $\vec{a} \| (\vec{b} - \vec{c})$ ...(ii)

From (i) and (ii), it follows that $\vec{b} = \vec{c}$, because $\vec{a}$ cannot be both parallel and perpendicular to vectors $(\vec{b} - \vec{c})$

31. We have, x = sint and y = sin pt,

$\therefore \frac{dx}{dt} = \cos t$ and $\frac{dy}{dt} = \cos pt.p$

$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{p.\cos pt}{\cos t}$ ...(i)

Again, differentiating both sides w.r.t. x, we get

$\frac{d^2y}{dx^2} = \frac{\cos t.\frac{d}{dt}(p.\cos pt)\frac{dt}{dx} - p\cos pt.\frac{d}{dt}\cos t.\frac{dt}{dx}}{\cos^2 t}$

$= \frac{[\cos t.p.(-\sin pt).p - p\cos pt.(-\sin t)]\frac{dt}{dx}}{\cos^2 t}$

$= \frac{[-p^2 \sin pt.\cos t + p\, sint.cospt].\frac{1}{\cos t}}{\cos^2 t}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-p^2 \sin pt.\cos t + p\cos pt.\sin t}{\cos^3 t}$ ...(ii)

Since, we have to prove

$\left(1-x^2\right) \frac{d^2 y}{d x^2}-x \frac{d y}{d x}+p^2 y=0$

$\therefore L H S=\left(1-\sin ^2 t\right) \frac{\left[-p^2 \, sinpt. \, cost+p \, cospt.\sin t\right]}{\cos ^3 t}$

$-\, sint. \frac{p \cos pt}{\cos t}+p^2 \sin pt$

$=\frac{1}{\cos ^3 t}\left[\begin{array}{c}\left(1-\sin ^2 t\right)\left(-p^2 \sin pt. \cos t+p \cos pt. \sin t\right) \\ -p \cos pt. \sin t. \cos ^2 t+p^2 \sin pt. \cos ^3 t\end{array}\right]$

$=\frac{1}{\cos ^3 t}\left[\begin{array}{l}-p^2 \sin pt. \cos ^3 t+p \cos pt. \sin t. \cos ^2 t \\ -p \cos pt. \sin t. \cos ^2 t+p^2 \sin pt. \cos ^3 t\end{array}\right]\left[\because 1-\sin ^2 t=\cos ^2 t\right]$

$=\frac{1}{\cos ^3 t} .0$

= 0 Hence proved.

Section D

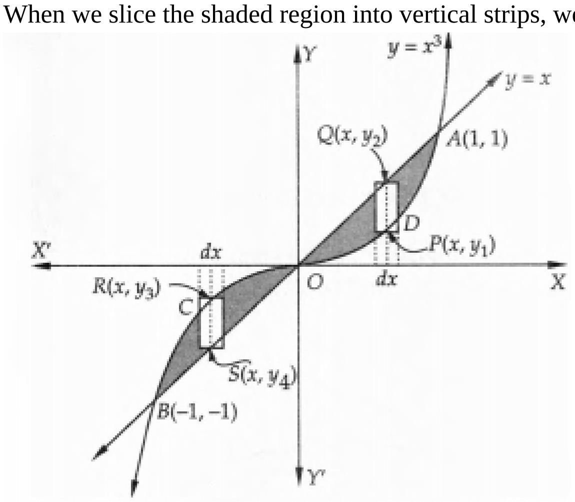
32. The given curves are ,

y = x ...(i)

and y = x^3 ...(ii)

The sketch of the curve y = x^3 is shown in Fig. Clearly, y =x is a line passing through the origin and making an angle of 45° with x-axis. The shaded portion shown in Fig. is the region bounded by the curves y = x and y = x^3. Solving y = x and y = x^3 simultaneously, we find that the two curves intersect at O (0, 0), A (1,1) and B (- 1, - 1).

When we slice the shaded region into vertical strips, we observe that the vertical strips change their character at O.



Therefore, the required area is given by,

Required area = Area BCOB + Area ODAO

Area BCOB: Each vertical strip in this region has its lower end on y = x and the upper end on y = x^3. Therefore, the approximating rectangle shown in this region has length $=\left|y_4-y_3\right|$, width = dx and area $=\left|y_4-y_3\right|$ dx. Since the approximating rectangle can move from x = - 1 to x = 0.

$\therefore$ Area BCOB $=\int_0^0\left|y_4-y_3\right| d x=\int_{-1}^0-\left(y_4-y_3\right) d x$ [$\because$ $y_4 < y_3$ $\therefore$ $y_4 - y_3 < 0$]

$=\int_{-1}^0-\left(x-x^3\right) d x$ [$\because$ R (x, y_3) and S (x, y_4) lie on (ii) and (i) respectively $\therefore$ $y_3 = x_3$ and y_4 = x]

$=\int_{-1}^0\left(x^3-x\right) d x=\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{-1}^0=0-\left(\frac{1}{4}-\frac{1}{2}\right)=\frac{1}{4}$ sq. units

Area ODAO: Each vertical strip in this region has its two ends on (ii) and (i) respectively. So, the approximating rectangle shown in this region has length $=\left|y_2-y_1\right|$, width = dx and therefore,we have,

Area ODAO $=\int_0^1\left|y_2-y_1\right| d x=\int_0^1\left(y_2-y_1\right) d x$ [$\because$ $y_2 > y_1$ $\therefore$ $y_2 - y_1 > 0$]

$=\int_0^1\left(x-x^3\right) d x$ [$\because$ P (x, y_1) and Q (x, y_2) lie on (ii) and (i) respectively $\therefore$ $y_1 = x^3$ and y_2 = x]

$=\left[\frac{x^2}{2}-\frac{x^4}{4}\right]_0^1=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$ sq. units

$\therefore$ Required area = Area BCOB + Area ODAO $=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$ sq. units

33. R = {(a,b) = |a.b| is divisible by 2.

where $a, b \in A=\{1,2,3,4,5\}$

reflexivty

For any a∈A,|a−a|=0 Which is divisible by 2.

$\therefore$ (a, a) ∈ r for all a ∈ A

So ,R is Reflexive
Symmetric :
Let $(a, b) \in R$ for all a, b ∈ R
|a−b| is divisible by 2
|b−a| is divisible by 2
$(a, b) \in r \Rightarrow (b, a) \in R$
So, R is symmetirc .
Transitive :
Let $(a, b) \in R$ and $(b, c) \in R$ then
$(a, b) \in R$ and $(b, c) \in R$
|a−b| is divisible by 2
|b−c| is divisible by 2
Two cases :
Case 1:
When b is even
$(a, b) \in R$ and $(b, c) \in R$
|a−c| is divisible by 2
|b−c| is divisible by 2
|a−c| is divisible by 2
∴ (a, c) ∈ R
Case 2:
When b is odd
$(a, b) \in R$ and $(b, c) \in R$
|a−c| is divisible by 2
|b−c| is divisible by 2
|a−c| is divisible by 2
Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
So R is transitive.
Hence , R is an equivalence relation

OR

Given that A = [-1, 1]

i. $f(x) = \frac{x}{2}$
   Let $f(x_1) = f(x_2)$
   $\Rightarrow \frac{x_1}{2} = \frac{x_2}{2} \Rightarrow$ $x_1 = x_2$
   So, f(x) is one-one.
   Now, let $y = \frac{x}{2}$
   $\Rightarrow x = 2y \notin A, \ \forall y \in A$
   As for $y = 1 \in A, \ x = 2 \notin A$
   So, f(x) is not onto.
   Also, f(x) is not bijective as it is not onto.
ii. g(x) = |x|
   Let $g(x_1) = g(x_2)$
   $\Rightarrow |x_1| = |x_2| \Rightarrow x_1 = \pm x_2$
   So, g(x) is not one-one.
   Now, x=±y $\notin A$ for all y ∈ R
   So, g(x) is not onto, also, g(x) is not bijective.
iii. h(x) = x|x|
   $\Rightarrow x_1|x_1| = x_2|x_2| \Rightarrow x_1 = x_2$
   So, h(x) is one-one
   Now, let y = x|x|
   $\Rightarrow y = x^2 \in A, \forall x \in A$
   So, h(x) is onto also, h(x) is a bijective.

iv. $k(x) = x^2$

Let $k(x_1) = k(x_2)$

$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

Thus, k(x) is not one-one.

Now, let $y = x^2$

$\Rightarrow x\sqrt{y} \notin A, \forall y \in A\ x = \sqrt{y} \notin A, \forall y \in A$

As for y = -1, $x = \sqrt{-1} \notin A$

Hence, k(x) is neither one-one nor onto.

34. On adding

$$5x + 5y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$5(x + y) = \begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix}$$

$$(x + y) = \frac{1}{5}\begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix} \text{------(i)}$$

On subtracting

$$x - y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \text{-------(ii)}$$

$$x - y = \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix}$$

Adding (i) and (ii) gives,

$$2x = \begin{bmatrix} \frac{4}{5} & \frac{-24}{5} \\ \frac{-22}{5} & 6 \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix}$$

$$x + y = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix} + y = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & \frac{1}{1} \end{bmatrix}$$

$$y = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & 1 \end{bmatrix} + \begin{bmatrix} \frac{-2}{5} & \frac{12}{5} \\ \frac{11}{5} & -3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{5} - \frac{2}{5} & \frac{1}{5} + \frac{12}{5} \\ \frac{3}{5} + \frac{11}{5} & 1 - 3 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

35. Let 1^{st} length = x

2^{nd} length = 28 - x

Now circumference of circle is $2\pi r$

$\therefore 2\pi r = x$

$\Rightarrow r = \frac{x}{2\pi}$

Now perimeter of rectangle = 4a

$\therefore 4a = 28 - x$

$\Rightarrow a = 7 - \frac{x}{4}$

ATQ

A = area of circle + area of square

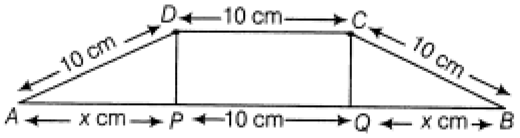
$$\pi\left(\frac{x}{2\pi}\right)^2 + \left(7 - \frac{x}{4}\right)^2$$

Now, $A = \pi . \frac{x^2}{4\pi^2} + \left(7 - \frac{x}{4}\right)^2$

So, $\frac{dA}{dx} = \frac{2x}{4\pi} + 2\left(7 - \frac{x}{4}\right)\left(-\frac{1}{4}\right)$

$\frac{dA}{dx}=0$
$\Rightarrow \frac{2x}{4\pi}+2\left(7-\frac{x}{4}\right)\left(-\frac{1}{4}\right)=0$
$\Rightarrow \frac{1}{2}\left(7-\frac{x}{4}\right)=\frac{x}{2\pi}$
$\Rightarrow 7-\frac{x}{4}=\frac{x}{\pi}$
$\Rightarrow 7=\frac{x}{\pi}+\frac{x}{4}$
$\Rightarrow 7=x\left(\frac{4+\pi}{4\pi}\right)$
$\Rightarrow \frac{28\pi}{4+\pi}=x$
Now, $\frac{d^2y}{dx^2}=\frac{1}{2\pi}-\frac{1}{2}\left(\frac{-1}{4}\right)$
$=\frac{1}{2\pi}+\frac{1}{8}$
positive, hence minimum
Therefore, 1^{st} length $=\frac{28\pi}{4+\pi}$
2^{nd} length $=\frac{28}{1}-\frac{28\pi}{4+\pi}$
$=28\left[\frac{4+\pi-\pi}{4+\pi}\right]$
$=\frac{112}{4+\pi}$

OR



Let ABCD be the given trapezium in which AD = BC = CD = 10 cm.
Draw a perpendicular DP and CQ on AB. Let AP = x cm
In ΔAPD & ΔBQC,
$\angle APD=\angle BQC$ [each =90°]
AD = BC [Both 10 cm]
DP = CQ [Perpendicular between parallel lines are equal in length]
$\therefore \quad \Delta APD \cong \Delta BQC$ [RHS Congruency]
$\therefore QB=AP$ [CPCT]
$\Rightarrow QB=x\ cm$
$DP=\sqrt{10^2-x^2}$ [by Pythagoras theorem]
Now, area of trapezium,
$A=\frac{1}{2}\times(\text{ sum of parallel sides })\times height$
$=\frac{1}{2}\times(2x+10+10)\times\sqrt{100-x^2}$
$=(x+10)\sqrt{100-x^2}$.....(i)
We need to find the area of trapezium when it is maximum i.e. we need to maximize area.
On differentiating both sides of eq(i) w.r.t.x, we get
$\frac{dA}{dx}=(x+10)\frac{(-2x)}{2\sqrt{100-x^2}}+\sqrt{100-x^2}$
$=\frac{-x^2-10x+100-x^2}{\sqrt{100-x^2}}$
$=\frac{-2x^2-10x+100}{\sqrt{100-x^2}}$ ....(ii)
For maxima or minima, put $\frac{dA}{dx}=0$
$\Rightarrow \frac{-2x^2-10x+100}{\sqrt{100-x^2}}=0$
= -2 (x^2 + 5x - 50) =0
= -2 (x+10) (x - 5) =0
x = 5 or -10
Since, x represents distance, so it cannot be negative.
Therefore, we take x = 5.

On diffentiating both sides of eq.(ii) w.r.t.x, we get

$\frac{\mathrm{d}^2 A}{\mathrm{d}x^2}=\frac{\sqrt{100-x^2}\cdot\frac{d}{dx}(-2x^2-10x+100)-(-2x^2-10x+100)\frac{d}{dx}(\sqrt{100-x^2})}{(\sqrt{100-x^2})^2}$ [by using the quotient rule of derivative]

$=\frac{\sqrt{100-x^2}\cdot(-4x-10)-(-2x^2-10x+100)(\frac{-2x}{2\sqrt{100-x^2}})}{(\sqrt{100-x^2})^2}$

$=\frac{\sqrt{100-x^2}\cdot(-4x-10)+\frac{x(-2x^2-10x+100)}{\sqrt{100-x^2}})}{100-x^2}$

$=\frac{(100-x^2)\cdot(-4x-10)+x(-2x^2-10x+100)}{(100-x^2)^{\frac{3}{2}}}$

$=\frac{-400x+4x^3-1000+10x^2-2x^3-10x^2+100x}{(100-x^2)^{\frac{3}{2}}}$

$\therefore\frac{\mathrm{d}^2 A}{\mathrm{d}x^2}=\frac{2x^3-300x-1000}{(100-x^2)^{3/2}}$

When x = 5,

$\frac{\mathrm{d}^2 A}{\mathrm{d}x^2}=\frac{2(5)^3-300(5)-1000}{\left[100-(5)^2\right]^{3/2}}$

$=\frac{250-1500-1000}{(100-25)^{3/2}}=\frac{-2250}{75\sqrt{75}}<0$

$\therefore$ It is maximum when x = 5

Thus, area of trapezium is maximum at x = 5 and maximum area is

$A_{\max}=(5+10)\sqrt{100-(5)^2}$ [put x = 5 in Eq. (i)]

$=15\sqrt{100-25}=15\sqrt{75}=75\sqrt{3}\text{ cm}^2$

Section E

36. i. P $\left(\frac{L}{C}\right)$ = $\frac{17}{100}$

ii. P $\left(\frac{\bar{L}}{A}\right)$ = 1 - P $\left(\frac{L}{A}\right)$ = 1 - $\frac{24}{100}$ = $\frac{76}{100}$ or $\frac{19}{25}$

iii. P $\left(\frac{A}{L}\right)$ = $\frac{\frac{1}{4}\times\frac{24}{100}}{\frac{1}{4}\times\frac{24}{100}+\frac{1}{4}\times\frac{22}{100}+\frac{1}{4}\times\frac{17}{100}+\frac{1}{4}\times\frac{9}{100}}$ = $\frac{24}{72}=\frac{1}{3}$

Probability that a randomly selected child is left-handed given that exactly one of the parents is left-handed.

= P $\left(\frac{L}{B\cup C}\right)$ = $\frac{22}{100}$ + $\frac{17}{100}$ = $\frac{39}{100}$

37. i. The line along which motorcycle A is running, $\vec{r}=\lambda(\hat{i}+2\hat{j}-\hat{k})$, which can be rewritten as

$(x\hat{i}+y\hat{j}+z\hat{k})=\lambda\hat{i}+2\lambda\hat{j}-\lambda\hat{k}$

$\Rightarrow$ x = λ, y = 2λ, z = -λ $\Rightarrow$ $\frac{x}{1}=\lambda,\frac{y}{2}=\lambda,\frac{z}{-1}=\lambda$

Thus, the required cartesian equation is $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$

ii. Clearly, D.R.'s of the required line are < 1, 2, -1 >

$\therefore$ D.C.'s are

$(\frac{1}{\sqrt{1^2+2^2+(-1)^2}},\frac{2}{\sqrt{1^2+2^2+(-1)^2}},\frac{-1}{\sqrt{1^2+2^2+(-1)^2}})$

i.e., $(\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}},\frac{-1}{\sqrt{6}})$

iii. The line along which motorcycle B is running, is $\vec{r}=(3\hat{i}+3\hat{j})+\mu(2\hat{i}+\hat{j}+\hat{k})$, which is parallel to the vector $2\hat{i}+\hat{j}+\hat{k}$.

$\therefore$ D.R.'s of the required line are (2, 1, 1).

OR

Here, $\vec{a}_1=0\hat{i}+0\hat{j}+0\hat{k},\vec{a}_2=3\hat{i}+3\hat{j},\vec{b}_1=\hat{i}+2\hat{j}-\hat{k}$, $\vec{b}_2=2\hat{i}+\hat{j}+\hat{k}$

$\therefore\vec{a}_2-\vec{a}_1=3\hat{i}+3\hat{j}$

and $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&-1\\2&1&1\end{vmatrix}$ = $3\hat{i}$ - $3\hat{j}$ - $3\hat{k}$

Now, $(\vec{a}_2-\vec{a}_1)\cdot\left(\vec{b}_1\times\vec{b}_2\right)=(3\hat{i}+3\hat{j})\cdot(3\hat{i}-3\hat{j}-3\hat{k})$

= 9 - 9 = 0

Hence, shortest distance between the given lines is 0.

38. i.

Corner Points	**Value of Z = 4x - 6y**
(0, 3)	$4 \times 0 - 6 \times 3 = -18$
(5, 0)	$4 \times 5 - 6 \times 0 = 20$
(6, 8)	$4 \times 6 - 6 \times 8 = -24$
(0, 8)	$4 \times 0 - 6 \times 8 = -48$

Minimum value of Z is - 48 which occurs at (0, 8).

ii.

Corner Points	**Value of Z = 4x - 6y**
(0, 3)	$4 \times 0 - 6 \times 3 = -18$
(5, 0)	$4 \times 5 - 6 \times 0 = 20$
(6, 8)	$4 \times 6 - 6 \times 8 = -24$
(0, 8)	$4 \times 0 - 6 \times 8 = -48$

Maximum value of Z is 20, which occurs at (5, 0).

iii.

Corner Points	**Value of Z = 4x - 6y**
(0, 3)	$4 \times 0 - 6 \times 3 = -18$
(5, 0)	$4 \times 5 - 6 \times 0 = 20$
(6, 8)	$4 \times 6 - 6 \times 8 = -24$
(0, 8)	$4 \times 0 - 6 \times 8 = -48$

Maximum of Z - Minimum of Z = 20 - (-48) = 20 + 48 = 68

OR

The corner points of the feasible region are O(0, 0), A(3, 0), B(3, 2), C(2, 3), D(0, 3).